

A new ontology for numerical cognition: Integrating evolutionary, embodied, and data informatics approaches

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ABSTRACT

Numerical cognition is a field that investigates the sociocultural, developmental, cognitive, and biological aspects of mathematical abilities. Recent findings in cognitive neuroscience suggest that cognitive skills are facilitated by distributed, transient, and dynamic networks in the brain, rather than isolated functional modules. Further, research on the bodily and evolutionary bases of cognition reveals that our cognitive skills harness capacities originally evolved for action and that cognition is best understood in conjunction with perceptuomotor capacities. Despite these insights, neural models of numerical cognition struggle to capture the relation between mathematical skills and perceptuomotor systems. One front to addressing this issue is to identify building block sensorimotor processes (BBPs) in the brain that support numerical skills and develop a new ontology connecting the sensorimotor system with mathematical cognition. BBPs here are identified as sensorimotor functions, associated with distributed networks in the brain, and are consistently identified as supporting different cognitive abilities. BBPs can be identified with new approaches to neuroimaging; by examining an array of sensorimotor and cognitive tasks in experimental designs, employing data-driven informatics approaches to identify sensorimotor networks supporting cognitive processes, and interpreting the results considering the evolutionary and bodily foundations of mathematical abilities. New empirical insights on the BBPs can eventually lead to a revamped embodied cognitive ontology in numerical cognition. Among other mathematical skills, numerical magnitude processing and its sensorimotor origins are discussed to substantiate the arguments presented. Additionally, an fMRI study design is provided to illustrate the application of the arguments presented in empirical research.

To understand how we learn and do math, we develop models of how different math-specific and domain-general processes work together to support numerical skills. Behavioral and neuroimaging studies experimentally test the validity of these models. Neuroimaging studies explore neural correlates; how biological systems support numerical skills. One fundamental assumption that underlie neuroimaging studies is that mathematical skills, and more generally cognitive skills, can be construed as collections of relatively simple processes—each associated with specific functions and localized brain networks. The idea of functional modules in the brain has been around for a while and can be traced back to the *computational theory of mind* (CTM) (Neumann, 1958; Rescorla, 2020). Historically, these functional modules had not been defined based on the study of biological systems, or more broadly brain-body-environment interactions. Instead, they were formulated based on the idea that the human mind operates similar to a computer, with functional modules found in computing systems. This approach has been

adopted in neuroimaging studies after the invention of fMRI and is still applicable today (Poldrack, 2010).

The current models in numerical cognition are constrained by what we assume are the basic functions and processes. A particular challenge with these models is their deficiency in explaining how number processing relates to bodily systems and skills. Traditional cognitive terms are inadequate in describing the interactions of brain-body-environment systems in situated contexts (Beer, 2008). One potential solution is to construct a novel cognitive ontology; one, where the cognitive concepts emerge from the study of the biological systems during authentic tasks, rather than being delineated by decontextualized computational functions. A cognitive ontology is described as a set of cognitive processes and their interrelationships (Hastings et al., 2014). What I call *building block processes* (BBPs) are fundamental sensorimotor functions that originally (in evolutionary terms) facilitated perceptuomotor skills and were repurposed to serve cognitive skills. These processes work together

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in transient and dynamic ways to support complex cognitive tasks.

In the last two decades cognitive neuroscience research has shifted its focus to the study of functional and structural networks in the brain. Network neuroscience research has revealed dynamic and distributed networks across different domains of cognition, yet we neither fully understand what these networks do, nor do we have a taxonomy categorizing them (Uddin et al., 2023). Previous efforts towards constructing alternative ontologies in neuroimaging focused on using large-scale data and data-analytics methods to study brain networks and the associated functions (Hastings et al., 2014; Passingham, Stephan, & Kötter, 2002; Poldrack & Yarkoni, 2016). Accumulation of work along these lines can eventually help build a comprehensive cognitive ontology of sensorimotor networks matching with tasks and behaviors. The theory of *neural use* can help with the interpretation of how these sensorimotor networks are associated with cognitive skills. This theory suggests that, during the evolutionary process, bodily systems and functions were repurposed to support complex cognitive functions (Anderson, 2014). Neural reuse can be considered a specific case of *exaptation*, which describes how a faculty previously serving a specific function gets co-opted to realize a new function, during the evolutionary process (Gould & Vrba, 1982). It is probable that BBPs draw heavily from basic sensorimotor operations that have been co-opted to aid cognitive functions. Cognitive skills like reading and mathematics are products of both biological and cultural evolution (Geary, 2007) and tracing the BBPs that underlie cognitive skills can only be possible by incorporating evolutionary perspectives when interpreting neuroimaging data.

The motivation for this paper is to discuss how we can develop a new cognitive ontology in numerical cognition guided by embodied and evolutionary theories and findings, and emerging data analytics approaches in neuroimaging. To set the stage, we first reflect on the history and current state of cognitive models in numerical cognition and discuss the limitations of incorporating findings on the bodily origins of mathematical skills in aggregate models. Then, magnitude representation and verbal encoding in numerical cognition are presented as case studies, and alternative interpretations are discussed based on embodied and evolutionary perspectives. Finally, a hypothetical neuroimaging study is presented to substantiate the arguments made.

1. The impact of the information processing approach on numerical cognition

The goal of numerical cognition as a field is to explain numerical skills, both in humans and other animal species, across the sociocultural, developmental, behavioral, cognitive, and biological (i.e., neural and genetic) levels. The origins of numerical cognition research can be traced back to clinical studies in the late 19th and early 20th centuries, when observations were made on how the numerical skills of patients were affected by neuropsychological conditions, induced by strokes, lesions, and injuries to the brain (Henschen, 1919; Rourke & Conway, 1997). Around the same time, psychologists studied mathematical thinking and methods for teaching mathematics (Thorndike, 1922). The thought processes that lead to mathematical insights were also a topic of interest (Hadamard, 1945). In the early 20th century, the emergence of behaviorism in the USA transformed the landscape of psychological research. Behaviorism emerged as a reaction to the use of introspection and mental states in psychology and targeted making psychology similar to natural sciences, solely relying on the study of observable behavior and stimulus-response relations (Watson, 1913).

It wasn't until after the second world war, when the study of the mind, mental states, and cognitive representations became the center of focus in psychology, during what we now call the *cognitive revolution* of the 1950s (Gardner, 1987). Fueled by excitement on what can be accomplished with computation and a push against behaviorism, ideas on natural cognition and artificial intelligence merged, yielding to the notion that cognition is a form of computation (Miller, 2003). Turing's abstract formulation of computation was followed by designs for

physical computers (Godfrey & Hendry, 1993). These developments coincided with discoveries on neuron function and their relation to learning and cognition (e.g., Hebb, 1949), propelling the idea that a neuron is a type of computational unit, with the spiking action indicating an all or none output based on the inputs. The neuron's method of calculating inputs and producing a binary output was comparable to the workings of digital computers, fostering the notion that the brain functions similarly to a computer.

Two pioneers of the cognitive revolution, Warren S. McCulloch and Walter Pitts developed a formal model of the brain as a network of neurons, each neuron acting as a simple logical processing unit, and networks of neurons together capable of carrying out logical operations (Arbib, 2000; McCulloch & Pitts, 1943). McCulloch (1961) was particularly interested in humans' numerical abilities: In his essay, titled "What is a number that a man may know it and a man that he may know a number?," he explained how an interconnected set of logical units can realize different numerical operations. The question of how the brain supports number processing, and cognition in general, then became a matter of the processing capabilities of different organizations of networks of neurons. This approach eventually led to the emergence of connectionism, which involves modeling of cognition with artificial neural networks (Rumelhart & McClelland, 1986).

Theories of computation distinguish between the hardware components (some form of a computer; a Turing machine) and the software; instructions outlining the logical processes taking place in an information processing system (Marr, 1982). The idea that computational or algorithmic processes were separate from the physical systems executing the computation implied that cognition could be studied in isolation from its biological correlates. Paralleling this view, Newell and Simon (1976) argued that "a physical symbol system has the necessary and sufficient means for general intelligent action" (p. 116). Here, a "physical symbol system" denotes a system capable of representing and processing expressions composed of symbols. This statement has two implications: first, that any sufficiently complex physical symbol system can yield intelligence; and second, that the human mind can be considered a physical symbol system. This approach underestimates the importance and the unique contributions of the human (and generally any) brain and body in cognition and approaches the mind as a symbol manipulation system. In a sense, the brain is treated as yet another generic computer, making the *physical symbol system* or the algorithm of the human mind the focus of investigation. With this approach, the field of cognitive science flourished, with studies experimentally testing to what extent the proposed cognitive models were able to predict behavioral data. In a nutshell, the human brain was reduced to a generic computer and the body to a trivial mass of flesh, leaving the abstract notion of the *mind* as the main focus of investigation.

Developments in imaging technologies, increased availability of computational power, and the motivation to better understand the neural correlates of cognition triggered the emergence of *cognitive neuroscience* as a field (Reuter-Lorenz, Baynes, Mangun, & Phelps, 2010). During the 1990s, cognitive neuroimaging focused on identifying brain areas associated with various cognitive processes previously studied in cognitive science. There has been early calls highlighting that complex cognitive functions are carried out by networks and not individual regions (Mesulam, 1990) and since 2000s, there has been a shift from matching specific functions with isolated brain regions to studying dynamic and distributed networks in the brain that support cognitive functions. Despite this shift, the theoretical frameworks in cognitive neuroscience are still deeply rooted in the cognitive models that were proposed decades before. Cognitive functions suggested by the computational theory of mind might not *actually* exist in the brain and capacities matching with these functions could be the result of multiple intertwined BBPs that we don't yet fully understand. In order to characterize these fundamental processes, we should consider not just the human mind and the brain, but the biological organism as a whole, its evolutionary past, its interactions with the environment, and the brain's

role in mediating these interactions.

1.1. Cognitive neuroscience research on numerical cognition

Cognitive neuroscience research on numerical cognition primarily examines neural correlates of number skills and individual differences. In addition to domain general processes (e.g., working memory, visuospatial abilities), numerical cognition focuses on domain-specific processes that underlie numerical skills; for example, number sense, counting, numerical magnitude representation, arithmetic calculation, and fact retrieval. Aggregate models of numerical cognition provide a map of cognitive processes matching with brain regions and synthesize findings across many studies to explain how number processing takes place in the brain (e.g., Arsalidou, Pawliw-Levac, Sadeghi, & Pascual-Leone, 2018; Faye et al., 2019; Menon, 2014).

Beyond mapping cognitive processes with brain regions, numerical cognition contributed to our understanding of how different phenomena, like age, existence of cognitive disorders or sociocultural factors, modulate neural underpinnings of numerical cognition. Thanks to this body of research, we now have a better understanding of the development of number sense in early childhood, the development of counting and arithmetic abilities in early years of schooling, and how early indicators (e.g., counting, working memory, verbal abilities, the home numeracy environment) predict later mathematical success (Hawes, Sokolowski, Ononye, & Ansari, 2019). Numerical cognition research also had an impact on mathematics education and classroom practices (Simplicio et al., 2020).

A subset of research in numerical cognition explores the relation between the bodily systems and measures, and number processing skills from an embodied cognition perspective. Embodied cognition, also known more broadly as 4E (embodied, embedded, enactive, and extended) cognition, serves as an umbrella term for a multitude of theories on how cognition is grounded in bodily systems and interactions. The common denominator of these theories is the principle that cognition cannot be understood merely as a mental phenomenon, and instead we should approach cognition as an emergent theme in how the brain and the body interact in physical and sociocultural contexts (Gallagher, 2017; Newen, De Bruin, & Gallagher, 2018). Research on embodiment of mathematical cognition (commonly referred to as *embodied numerosity* or *embodied mathematics*) examines the connection between bodily skills and measures (such as gestures, fine- and gross-motor abilities, and finger counting) and numerical abilities; how bodily measures are linked to and can predict these abilities; how these relationships manifest at the neural level; and how situated activity in physical and social contexts can facilitate learning.

The embodied perspective asserts that, instead of assisting the development of abstract mathematical representations and principles, bodily interactions structure the sensorimotor system in order to provide semantic content for number processing (Abrahamson, 2021; de Freitas & Sinclair, 2013; Lindemann & Fischer, 2015; Soylu, 2016). This body of research moves towards understanding numerical cognition not merely as a computational process confined to the brain, but as a theme emerging from the interactions among the brain, body, and environment—within cultural and evolutionary contexts.

2. The body and mathematics relation: evolutionary and embodied perspectives

Mathematics is a distinct human capability that has emerged, in part, thanks to recent human cultural evolution. Geary distinguishes between two types of number skills: 1) *Biologically-primary* skills are hereditary and not dependent on enculturation. Some of these skills—such as approximate numbers skills—are shared with other animals (Brannon & Merritt, 2011). 2) *Biologically-secondary* skills, on the other hand, are developmental, show cultural variance, and are results of human cultural evolution. Symbolic representations of numbers and number

operations are some examples.

The vast diversity in how human cultures represent and communicate numbers, combined with the minimal evolutionary changes in humans over the last several thousand years since we started using symbolic numbers, strongly suggest that symbolic mathematics is primarily a product of cultural evolution (O'Shaughnessy, Gibson, & Piantadosi, 2021). This also suggests that mathematical cognition represents a case of *neural reuse* and mathematical abilities make use of biological systems that originally evolved to support other functions (Anderson, 2016). Therefore, it is essential that we adopt an evolutionary perspective in our efforts to make sense of human mathematical skills. The emergence of symbolic mathematical skills from some foundational sensorimotor skills is a story of both biological and cultural evolution.

Studies with a diverse group of vertebrates (e.g., chimps, mice, birds, fish) show that humans share some non-symbolic mathematical abilities with other species, including an object tracking system, which can distinguish quantities up to four, and an approximate number system, which can make approximate magnitude judgments on larger quantities (Geary, Berch, & Koepke, 2015). Human infants show similar capabilities (Jordan & Brannon, 2006) and these number skills are considered as part of the humans' biological readiness for mathematics. However, the ability to process large, exact, and symbolic numbers is thanks to our cultural evolution, evidenced by both the presence and absence of symbolic and exact numbers across cultures, myriad of different ways numbers are represented in different cultures, and the need for enculturation and learning opportunities for number skills to develop (Beller et al., 2018; O'Shaughnessy et al., 2021).

Our current efforts to understand the neural mechanisms supporting cognitive, bodily, and social skills need to take the intertwined coevolutionary history linking these skills. Human cognitive skills, like reading and symbolic mathematics, make use of sensorimotor systems that evolved to support other functions (Anderson, 2014, 2016). In the brain, these unique cognitive skills make use of large networks of distributed systems, working together in dynamic and transient ways (Sporns, 2011). Each brain region participates in many different networks and the role of a particular region in a single function alone is not sufficient to characterize it. For example, while it is established that angular gyrus (AG) activates during arithmetic retrieval (Grabner et al., 2009), it also participates in a wide range of tasks, including semantic processing, reading, visuospatial processing, social cognition, and finger individuation (Anema et al., 2008; Rusconi, Walsh, & Butterworth, 2005; Seghier, 2012). Similar to AG, most brain regions commonly included in neural models of mathematical cognition participate in a wide range of tasks, complicating attribution of functions to brain regions. Anderson (2007) explains this wider phenomenon with an apt analogy: "... finding the role of a given brain area will be something like finding the right letter to go into a box on a (multidimensional) crossword puzzle, determined not just by the answer to a single clue, but by all the clues whose answers cross that box" (p. 339). Like the letters in a crossword puzzle, each brain region contributes to multiple different functions. As is the case for AG, it is often not immediately clear how and why a specific region participates in multitude of networks, associated with seemingly unrelated tasks. Explaining the functional contribution of distinct systems to a wide variety of cognitive functions has to consider the history of how these systems evolved and how each cognitive function is associated with the building block sensorimotor processes (BBPs) involved.

What does it mean for sensorimotor processes to support cognition? Theories of embodied cognition (EC) are broadly categorized into two as *weak EC* and *strong EC* (Gallagher, 2017). Weak EC reduces situated action in the environment to sensorimotor processing in the brain and emphasizes body-based mental representations (e.g., Barsalou, 1999). On the other hand, strong EC theories view cognition as an emergent outcome of the intricate interaction between the brain, body, and environment (e.g., Chemero, 2011; Varela, Thompson, & Rosch, 1991).

This perspective asserts that cognition cannot be solely reduced to brain processes. While strong EC accounts aim to explain cognition without relying on mental representations, they are challenged by “abstract” domains of cognition, such as mathematical thinking, memory, reasoning, and mental imagination. Such *representation-hungry* forms of processing are *offline*; they take place in the absence of a physical context that is relevant to the subject of processing (Clark & Toribio, 1994). The term *representation-hungry* here refers to the difficulties of explaining some forms of cognition without resorting to disembodied mental representations.

Mental arithmetic, solving a chess puzzle, or retrieving a childhood memory are all processes that bear little or no relation to the ongoing interactions with the organism’s current context. Simulation accounts of cognition come to help in such cases, where offline processing is characterized as simulated sensorimotor processing (Gallese & Lakoff, 2005; Soylu, 2016; Svensson, Lindblom, & Ziemke, 2007). Simulation theories characterize offline processing as a form of internal sensorimotor activity. The relation between the body and number processes is more overt and observable early in the development, becoming progressively more offline, with number processes involving fewer interactions with the environment. For example, even though children rely on fingers, audible counting, or physical manipulatives to do arithmetic, adults mostly rely on some form of offline processing (e.g., mental calculation, retrieval). Mathematics becomes more decontextualized with development, making interactions with the environment less central. Nevertheless, the traces of those early bodily interactions can be observable in adulthood (Michaux, Masson, Pesenti, & Andres, 2013; Soylu & Newman, 2016). For example, across numerous studies, number processing tasks were found to activate sensorimotor areas associated with fingers and behavioral associations between finger (e.g., tapping, finger gnosis, fine-motor ability) and number measures were found (see Barrocas, Roesch, Gawrilow, & Moeller, 2020; Berteletti & Booth, 2016; Soylu, Lester, & Newman, 2018 for reviews).

3. What is missing in numerical cognition models?

Despite substantial evidence supporting the role of sensorimotor systems in numerical cognition (Sixtus, Krause, Lindemann, & Fischer, 2023), the existing neurocognitive models for number processing fall short in explaining how these sensorimotor systems actually facilitate numerical processes. As highlighted by Roesch and Moeller (2015), the deficiency in our present models of number processing becomes evident in how the research findings regarding the connection between finger sensorimotor abilities and number processing are frequently overlooked in the context of neurocognitive models of numerical cognition. This is not necessarily due to shortcomings in the proposed models themselves. Instead, the issue stems from the prevailing cognitive ontology that has held influence in the field of numerical cognition and more broadly in cognitive neuroscience.

To add more clarity to the argument here, let’s consider two brain regions commonly featured in numerical cognition models and their associated functions: the intraparietal sulcus (IPS) and the angular gyrus (AG). We will first focus on a fundamental capacity in numerical cognition, magnitude representation—which is often associated with IPS—and discuss how the relationship between magnitude representation and action, along with the associated sensorimotor systems, are addressed within neurocognitive models. Then, we will visit AG, the functions mapped to it in numerical cognition models, and discuss the more expansive roles of AG as an association area. Going back to the aforementioned puzzle analogy, we will discuss why we are challenged with getting the pieces of the puzzle right.

3.1. Magnitude processing and the IPS

Magnitude processing refers to the ability to estimate, represent, and process magnitudes across multiple modalities (mainly visual, but also

auditory, olfactory, etc.). A magnitude is the size or extent of a specific measure, for example the length of a rod, the distance to an object, the weight or volume of an object. Drawing from behavioral and neuroimaging studies, showing associations between non-numerical (physical) and numerical magnitude processing, it has been proposed that physical magnitude estimation is the precursor for numerical magnitude representation and that numerical magnitude processing makes use of systems originally evolved to process physical magnitudes, pointing to a domain-general magnitude processing system (Cohen Kadosh, Lammertyn, & Izard, 2008; Dormal, Andres, & Pesenti, 2012; Moyer & Landauer, 1967).

A key principle in embodied cognition is the inseparability of perception and action, underscoring the importance of understanding perception in the context of action. Prior research, encompassing diverse forms of magnitudes, indicates that when individuals are tasked with estimating a magnitude, their perception goes beyond merely recognizing an objective magnitude. Instead, they incorporate their own body, morphology, and the potential for actions within the environment into this estimation process. According to this action-specific account of perception, humans perceive the environment based on their ability to act in it (Gibson, 1979). For example, objects positioned to be easier to grasp are estimated to be closer (Linkenauger, Witt, Stefanucci, Bakdash, & Proffitt, 2009), graspable object sizes are judged relative to the hand size (Linkenauger, Leyrer, Bühlhoff, & Mohler, 2013), and object sizes are perceived relative to the body size (van der Hoort, Guterstam, & Ehrsson, 2011). These effects extend to estimations involving slope, volume, weight, time, distance, and other factors. Collectively, these findings illustrate that estimations across various magnitude categories are impacted by bodily capacities, morphology, and the perceived potential for actions (see Philbeck & Witt, 2015 for a review). Across these instances, magnitude processing and representation are intertwined with action possibilities rather than being solely concerned with pure magnitude estimations.

Neuroimaging studies have pointed to the involvement of bilateral parietal areas in the processing of numerical magnitudes, in particular the intraparietal sulcus (IPS) (Ansari, 2007; Faye et al., 2019). Besides neural overlap and associations among various forms of magnitude processing, neuroimaging studies also revealed distinct neural correlates for symbolic numerical, non-symbolic numerical, and non-numerical magnitude processing. For example, Fias, Lammertyn, Reynvoet, Dupont, and Orban (2003) identified a segment of the left IPS participating in both symbolic and non-symbolic magnitude comparison, in addition to an anterior segment that only participates in symbolic number comparisons. In a meta-analysis study, Sokolowski, Fias, Ononye, and Ansari (2017) reported more left-lateralized superior parietal involvement for symbolic numerical, compared to both non-symbolic numerical and non-numerical magnitude processing, and distinct activations in the bilateral precuneus for only non-symbolic numerical processing. While neuroimaging studies consistently indicate the involvement of the IPS in processing both physical and numerical magnitudes, IPS is not exclusively dedicated to magnitude processing.

Given the evidence for magnitude representation being intimately tied with action, particularly action with hands and fingers, it is reasonable to expect IPS to have a significant role in object-oriented actions. In fact, across both human and monkey studies, IPS is found to be responsible for integration of multimodal information, involving both proprioception, and planning and execution of object-oriented actions (e.g., grasping, object manipulation) (see Grefkes & Fink, 2005 for a review). Within the IPS, the anterior portion (aIPS) participates in the integration of visual information, encoding graspable features of objects, and target goal, and the posterior portion participating in the experiential knowledge of object-use to guide action (Mruczek, von Loga, & Kastner, 2013; Tunik, Ortigue, Adamovich, & Grafton, 2008). In a TMS study, the stimulation of the right aIPS was found to perturb grip aperture during bimanual action (Le, Vesia, Yan, Niemeier, & Crawford, 2014), pointing to the role of this region in adjusting the grip aperture

during the grasping action. The anterior and posterior parts of IPS were also reported to have different structural and functional connectivity profiles, the posterior section linking with the insula, premotor and middle frontal areas, while the posterior IPS links with extrastriate visual areas (Uddin et al., 2010). Further, in a large-scale meta-analysis study, Yeo et al. (2015) developed a map of functional specialization and flexibility for cortical regions, which pointed to IPS as one of the most functionally flexible (participating in a diverse range of functions) regions in the cortex. Both the functional and structural findings point to IPS as a region integrating visual, tactile, and motor modalities in the service of many different perceptuomotor and cognitive functions.

As mentioned above, prior research has reliably associated the IPS with both numerical and nonnumerical magnitude processing. Yet, when the broader literature is examined, it becomes evident that the IPS serves a wider function as an association area, and more specifically in facilitating visually coordinated manual actions. Describing the IPS as a magnitude processing module makes it challenging to position number processing within the broader context of biological systems that evolved to support action. Multiple studies on the relation between hands/fingers and numbers allude to the more widely defined role of IPS in visually coordinated manual action. For instance, Andres, Ostry, Nicol, and Paus (2008) observed that Arabic numerals written on physical objects influenced grip aperture during the initiation of grasping actions; with larger numerals accompanying larger grip apertures. This finding highlights the overlap and interference between numeric and physical magnitude representations during the grasping action. In a separate study, Badets, Andres, Di Luca, and Pesenti (2007) explored the interplay between estimation of *affordances* (object and organism specific action possibilities; Gibson, 1979) and numerical magnitude representation. In an initial experiment, the act of grasping rods was influenced by a preceding presentation of numbers. Small numbers led to overestimated grasps—subjects estimated they could grasp the rod when they couldn't—while large numbers resulted in underestimated grasps. In a subsequent experiment, participants judged whether two rods were of equal length, a task that did not involve grasping. Notably, the numerical primes did not have any effect on the judgment of physical size. Together, these experiments demonstrated that physical and discrete numerical magnitude systems overlap only in the context of the grasping movement.

3.2. The role of angular gyrus in numerical cognition

Much like magnitude processing, other processes incorporated into numerical cognition models, such as verbal processing and symbol (Arabic numeral) processing, can also be unraveled to delve into their sensorimotor and evolutionary origins. In the inferior parietal cortex, the left angular gyrus has been consistently found to be more active during arithmetic fact retrieval. Because angular gyrus is thought to be part of the *perisylvian language network*, this activation has been interpreted as verbal coding of numbers or retrieval of verbally stored facts (Dehaene, Piazza, Pinel, & Cohen, 2003; Delazer et al., 2003; Grabner et al., 2009). However, there are alternative models, for example, pointing to hippocampus and para-hippocampus for arithmetic retrieval, and associating angular gyrus with attention orientation (Bloechle et al., 2016).

Similar to IPS, outside of numerical cognition, AG is associated with many different functions, including semantic processing, conceptual integration, spatial coordination, memory retrieval, reading comprehension, the default mode network, attention, spatial cognition, theory-of-mind, and social cognition (see Igelström & Graziano, 2017; Seghier, 2012 for reviews). The inferior parietal cortex, housing the angular gyrus in its posterior region, stands out as one of the most prominent connection hubs in the brain (Tomasi & Volkow, 2011). Also, similar to IPS, AG shows dissociable functional and structural connectivity across its subregions; the anterior section is linked to basal ganglia, premotor and ventrolateral prefrontal areas, whereas the posterior section is

linked to regions that are part of the default-mode network (DMN), including the ventromedial prefrontal, posterior cingulate, para-hippocampal and hippocampal areas (Uddin et al., 2010). AG activates across a wide range of tasks, suggesting that understanding its function may require looking beyond the confines of a single domain. AG, being positioned in the temporoparietal junction, is adjacent to visual, spatial, and somatosensory association areas, and is highly connected to other parts of the brain, making it ideal for receiving inputs from multimodal areas and conducting heteromodal integration.

In a comprehensive meta-analysis encompassing various semantic tasks, Binder, Desai, Graves, and Conant (2009) identified the angular gyrus as a central hub consistently activating across diverse semantic tasks. This led to the proposition that the angular gyrus serves as a primary system for conceptual integration, playing a role in various forms of semantic processing. One approach linking sensorimotor and semantic processing suggests that the integration of multimodal information in the association areas is central to concept representations (Barsalou, 2008; Pulvermüller, 2005). The meaning of a concept comes from the context- and experience-dependent sensorimotor simulation of visual, tactile, and auditory modalities related to that concept (Gallese & Lakoff, 2005). Based on fMRI and DTI findings, AG is proposed to be a hub for the heteromodal conceptual processing system, which integrates content-specific information from multiple modalities for semantic memory and representation (Binder et al., 2009; Bonner, Peelle, Cook, & Grossman, 2013).

Apart from the extensive body of research in numerical cognition that links the AG to verbal retrieval of arithmetic facts, there exists a set of studies indicating that the AG serves as a convergence point between finger sensory processing and mathematical cognition. In 1924 Gerstmann diagnosed a patient who was not able to name or point to her fingers, due to a lesion in the left AG. Further tests showed that the patient also had difficulty differentiating between their own or another person's right and left hand, performed poorly in arithmetic tests, and showed difficulty in writing (Gerstmann, 1940). Similar findings have been replicated with patients with parietal lesions (Mayer et al., 1999). In two studies, stimulation of angular gyrus in healthy adults was shown to disrupt both finger sense and number processing (Roux, Boetto, Sacko, Chollet, & Tremoulet, 2003; Rusconi et al., 2005). In the only fMRI study on finger gnosis—a measure of finger sense, measuring the ability to localize the stimulation of fingers—the left inferior parietal lobule emerged as one of the parietal neural correlates, together with bilateral precuneus (Rusconi et al., 2014). The activation of the angular gyrus (AG) during finger gnosis aligns with the previously discussed heteromodal function. This is because the finger gnosis task necessitates the mental representation of a finger schema, involving the integration of visual, tactile, and motor modalities. The process overlap between number and finger sensorimotor processing in AG was shown in a dual-task study, where adult participants were asked to tap on their fingers and concurrently answer arithmetic questions. The results showed differential interference of anatomically ordered tapping on single-digit addition, compared to double-digit addition, with particularly left AG showing a higher process overlap between these two tasks. The results were interpreted as arithmetic fact retrieval activating finger representations, due to associations that were established during finger counting experiences in early childhood (Soyulu & Newman, 2016).

Overall, despite the empirical confirmation of the angular gyrus's involvement in arithmetical retrieval, its precise role remains ambiguous. The diverse heteromodal role of the angular gyrus (AG) across various task domains, coupled with the findings from prior numerical cognition studies that demonstrate an interaction between number and finger sensorimotor processing, suggests that the AG's involvement in number processing deserves a fresh look that goes beyond verbal memory retrieval. In fact, verbal processing itself is grounded in the sensorimotor system (Cappa & Pulvermüller, 2012), exemplified with effects like comprehension of action verbs (e.g., *kick*, *pick*) activating somatotopically relevant parts of the motor system (Pulvermüller, 2005).

and behavioral sensorimotor effects in processing of action verbs (Muraki, Dahm, & Pexman, 2023). Besides, a recent study provides evidence for what is called the *cortical language network* to be actually a consequence of averaging across diverse linguistic processes, leaving only cortical connectivity hubs at the aggregate, and masking more complex, dynamic, and distributed networks, not limited to the cortex (Aliiko, Wang, Small, & Skipper, 2023).

The AG's engagement in various tasks signifies its role as a hub for heteromodal integration, facilitating semantic processing and memory across diverse domains. AG's function is part of a larger question about the origins of semantic processing and how sensorimotor networks in the brain support semantic understanding and memory. A cognitive ontology that has a corpus of sensorimotor BBPs in relation to formation and retrieval of semantic memories could help explain how AG supports arithmetic memory retrieval and its relation to seemingly distant skills, like finger sensorimotor skills. We do not yet have such a cognitive ontology and building one will require empirical studies that can help unfold the BBPs connecting diverse domains of cognition.

4. How to develop a new cognitive ontology

4.1. Implementation of data analytics approaches

The move from focusing on regions to dynamic networks came with changes in data analysis methods. New approaches, such as multivariate pattern analysis (MVPA) or connectivity analysis, enabled study of many-to-many structure-function relationships and consideration of networks, rather than distinct regions, as functional units. How each region contributes to numerous functions can be tackled by gathering neuroimaging data involving a variety of tasks and examining the distinct contributions of each region to the array of tasks studied. In a pioneering study, Passingham et al. (2002) introduced the notions of *connectional* and *functional fingerprints*, which are multidimensional representations for functional profiles of different cortical regions. A connectional fingerprint represents the structural connectivity of a region, while a functional fingerprint represents the disposition of a region; the extent with which a region participates in a variety of tasks. Passingham et al. used perceptuomotor tasks across multiple modalities, such as auditory, visual, tactile, and proprioceptive, to characterize the functional contributions of each region in the cortex. The results showed that structural connectivity, to some degree, accounted for functional fingerprints of different brain areas. More importantly, this study introduced the structure-function match as a one-to-many relation, each region participating in multiple functions, across different scales.

Anderson, Kinnison, and Pessoa (2013) employed a similar methodology to that of Passingham et al. (2002) to study functional diversity of brain areas. However, in this case, they defined a functional fingerprint as a 20-dimensional vector, each dimension representing activation maps from different tasks, categorized under four categories: action (e.g., motor learning), cognition (e.g., working memory), emotion (e.g., fear), and perception (e.g., audition). With the functional fingerprints, they were able to show how much each region contributed to the different types of task categories, in a weighted manner. They also introduced a network measure, named *assortivity*, indicating the similarity of functional fingerprints across various regions that constitute a network. This approach enables a brain region to be characterized not by a single function, but by a multidimensional vector that portrays the region's weighted contribution to a spectrum of tasks.

While it is important to note that each region supports multiple different functions, it is the networks that carry out cognitive functions and not individual regions. In most cases, brain networks show *emergent* characteristics. *Emergence* here refers to new properties or behaviors observed when elements of a complex system interact; properties that cannot be observed when each part is studied in isolation. Pessoa (2022) calls this phenomenon *interactional complexity*, and makes a call for shifting our functional unit of analysis from brain regions to functional

networks. The functional networks in the brain are not mutually exclusive and one region participates in multiple networks, realizing different functions. In addition to individual brain regions exhibiting functional diversity (Anderson et al., 2013; Passingham et al., 2002), functional networks show a high-degree of overlap and they dynamically change depending on the task context (Najafi, McMenamin, Simon, & Pessoa, 2016). Analytical approaches, like network science, and dynamic and complex systems, have been implemented to characterize the different, resting-state and task-related networks commonly observed in the brain. However there has not yet been consensus on what constitutes a network, the taxonomy of networks, or the nomenclature used to describe networks within the brain (Uddin, 2023). Given the rapid pace of advancements in network neuroscience, it is likely that these issues will be addressed in the future.

4.2. Exploring sensorimotor functions through an evolutionary lens

The next challenge is how we define functions. Traditionally, cognitive functions—such as perception, attention, memory, and language—are defined in disembodied and computational terms. However, it is likely that the BBPs in the brain are different from those computational functions found in information processing models. As Poldrack (2010) exemplifies, "... while we think that 'working memory' is a unique function implemented in the brain, it may be the case that there is no such function implemented by the brain and that what we call *working memory* is in reality a combination of some other functions" (p. 755). What are these "other functions"? We don't yet know for sure; some recent work provides preliminary ideas on the nature of these *native*—as in emerging from the biological mechanisms; not conceived independent of biological correlates—functions. Anderson (2014) proposes sensorimotor primitives in the brain, named *neuroscientifically relevant psychological (NRP) factors*, which come together in dynamic ways to support cognitive abilities. Each NRP factor makes use of multiple regions and multiple NRP factors contribute to a given cognitive state. Illustrating a similar approach, Poldrack, Halchenko, and Hanson (2009) used a decoding method to classify fMRI data from eight tasks and used a dimension-reduction method to determine six dimensions (patterns of activation) that are most characteristic. These dimensions embody processes that are not determined a priori, but instead emerge from the analysis of data. Each dimension was found to have differential contributions to the eight tasks studied, showing both differences and similarities across the tasks. From a single study, it is not immediately clear what the functions of these dimensions are, however multiple studies using similar methods can help uncover the nature of these functions.

Pessoa, Medina, and Desfilis (2022) argue that "vertebrate neuro-architecture does not respect the boundaries of standard mental terms" and that neuroscience studies should instead focus on "the dynamic coupling between large-scale brain circuits and complex, naturalistic behaviours" (p. 1.). The *mental terms* mentioned are psychological constructs and domains such as cognition, emotion, attention, and memory. They advocate for neuroscience research to prioritize exploring the interplay between large-scale brain circuits and real-world behaviors. Their critique extends to several putative distinctions made, for example between cognition and affect, cortical and subcortical systems, and the additive nature of phylogenetically old and new systems in the brain. These arguments are supported by neuroscience studies showing that neural correlates of tasks associated with, for example, cognition and emotion involve intertwined dynamic and distributed networks, and that the modularity suggested by mental domains are not accommodated by neuroscience findings. The alternative path suggested is to focus on neural circuits subserving specific classes of behavior, while also considering the evolutionary contexts of these behaviors. Cisek (2022) similarly objects to putative mental terms and emphasizes the significance of evolutionary approaches in elucidating how brain systems facilitate contextual behaviors and skills. They propose a taxonomy

delineating the neural mechanisms governing basic sensorimotor control in vertebrates. This taxonomy traces phylogenetic progression across various species and examines how organism morphology, nervous systems, and behaviors have co-evolved to adapt to evolutionary pressures.

Even though we don't yet have comprehensive models of numerical cognition guided by the approaches discussed so far, there is some promising preliminary work. For example, [Vallortigara, Lorenzi, Mesina, and Perrino \(2022\)](#) combines evidence from a multitude of studies involving various vertebrates (e.g., fish, birds, primates) and proposes a model of continuous and discrete magnitude processing, involving both pallial (cortical in mammals) and subpallial (subcortical in mammals) systems. The model includes midbrain structures, thalamus, sensory pathways, and cortical association regions and explains how behaviors involving magnitude processing are supported by distributed networks, not limited to the cortex. The proposed approach holds promise in identifying core sensorimotor functions and associated neural circuitry, which can subsequently inform our understanding of human cognitive abilities, such as mathematics.

Based on the discussions so far, what type of numerical cognition neuroimaging study would allow exploring the BBPs in numerical cognition? Despite recent neuroimaging research showing that the neural correlates of cognitive processes are transient, dynamic, and distributed networks, research into functional connectivity has also indicated that fundamental sensory and motor functions exhibit a degree of specialization ([Yeo et al., 2011](#)). There is evidence suggesting that phylogenetically older functions (functions that emerged relatively early in evolution) tend to associate with more localized networks, whereas newer cognitive functions tend to involve more extensive networks ([Anderson, 2014](#)). In a sense, while the evolutionary brain building project favored specialized sensorimotor functions early on, the acquisition of newer functions—relying on diverse combination of these earlier functions—demanded further and more intricate interlinking of distinct brain systems. Mathematics, one of the most recent human cognitive abilities, is widely distributed and draws upon a variety of functions that originally evolved for different purposes. Large-scale projects involving many subjects, tasks, and modalities, such as the Human Connectome Project, drive many of the studies harnessing new data analytic approaches. Unfortunately, these projects do not typically include number processing (e.g., arithmetic, magnitude comparison) and relevant sensorimotor (e.g., finger tapping, gnosis) tasks. Therefore, it would be beneficial to explore what type of experimental data would allow the type of data-driven research discussed. In typical numerical cognition neuroimaging studies, tasks are often confined and centered on the specific skills under investigation. However, studies involving number tasks together with tasks that engage the relevant sensorimotor functions have the potential to yield fresh insights.

4.3. Charting the course for future research

How should we design an experimental paradigm and analyze data to explore the BBPs? What we call a BBP here is essentially a functional sensorimotor network that matches with a functional label that alludes to what this network does. Despite the proliferation of network neuroscience research, we do not have yet a taxonomy of functional networks, both because there is no consensus on what constitutes a network and because it is not clear what the networks that reliably emerge across different studies do ([Uddin, Yeo, & Spreng, 2019](#)), perhaps with a few exceptions such as the *salience* and *default-mode* networks ([Uddin et al., 2023](#)). However, there has been some promising examples for research on cognitive ontologies that can help chart the path ahead.

Machine learning methods, particularly decoding, have been increasingly used in cognitive neuroscience. Decoding involves training and testing of a machine learning algorithm to predict task conditions or mental states based on the neural data. This process involves training the algorithm with a set of data and testing it with another. The testing

indicates how well the trained algorithm captures the aspects of the signal that characterizes the differences among task conditions or mental states. Decoding methods have been shown to predict the task or mental states in fMRI experiments with high accuracy, across different domains. Decoding analysis provides different insights than the more typical parametric testing (e.g., GLM) with fMRI data; for instance, it shows whether the signal studied has information that characterizes the classes (conditions, mental states etc.) studied, which properties of the signal contributes towards the classification and how well the model developed generalize, with testing on new datasets ([Naselaris, Kay, Nishimoto, & Gallant, 2011](#)). Decoding analysis involves preprocessing of data, feature extraction, training of the classification algorithm, and testing of the trained algorithm by measuring classification performance with new data.

fMRI data comprises time-series data with an intricate spatial component, involving many voxels (up to about a million, depending on the resolution). Each voxel produces signal values across the time series. When employing machine learning algorithms for data analysis, each voxel serves as a feature. However, dealing with hundreds of thousands of features is not computationally viable. To address this issue, *feature extraction (dimension reduction)* methods are used, which condense the data into a smaller feature-set, without significantly compromising the ability to identify unique patterns associated with each task condition. Unsupervised feature extraction methods, such as principal component analysis (PCA) or independent component analysis (ICA), do not make use of the class labels when transforming the data to a reduced set of features. Conversely, supervised methods, like linear discriminant analysis (LDA), enable the selection of features crucial for distinguishing data from one class versus others.

In a decoding study including eight diverse tasks (e.g., gambling decisions, rhyme judgments, and working memory), [Poldrack et al. \(2009\)](#) used an entropy measure for feature selection, leading to the selection of ~2 K voxels with the highest entropy (variability across tasks), which were then used for a neural network decoding analysis. The neural network analysis produced comparable accuracy to a whole-brain support vector machines (SVM) analysis, showing that the reduced dataset with much fewer voxels included the information needed to classify the different tasks. They then used an agglomerative hierarchical clustering approach to extract neural activity patterns from each dimension of the reduced space formed by the hidden units of the neural network. A *dimensional loading* process was followed to determine to what extent each dimension matched with each task. To understand what the activity patterns for each dimension meant, in terms of the type of processing taking place, they matched each task with a collection of cognitive concepts from a basic cognitive ontology (e.g., language, audition, spatial processing).

[Walters et al. \(2022\)](#) used an fMRI dataset with 44 diverse task conditions and matched each condition with a set of perceptual-motor or cognitive ontological constructs from the Cognitive Atlas ([Poldrack et al., 2011](#)). They trained a cognitive encoding model with a subset of the tasks and found that the model generalized to untrained tasks. In other words, the data-driven ontology developed was able to predict activation patterns of new tasks and even of resting-state networks. This study is innovative in multiple ways. It shows that encoding can be used to assess cognitive theories. The predictive success of the encoding model based on the cognitive theory provides a new way of assessing cognitive theories. This approach also can help with refining a cognitive ontology: If the inclusion of an ontological construct consistently fails to improve prediction results, this might show that the construct is either ill-defined, largely overlaps with other constructs, or the tasks associated with the construct are not in fact associated.

Both [Poldrack et al. \(2009\)](#) and [Walters et al. \(2022\)](#) show how paradigms with diverse tasks can help characterize networks that contribute to a wide-range of cognitive concepts. Furthermore, they show that decoding and encoding can be complementary in exploring new networks and functions ([Naselaris et al., 2011](#)), and how to validate

these, as part of the mission to develop a cognitive ontology. To illustrate the practical application of the ideas outlined here, let's explore a hypothetical study design aimed at investigating the relation between the hand-finger sensorimotor system and numerical cognition. Building upon the previously discussed link between numerical magnitude processing and object-oriented action affordances, a study involving different forms of magnitude tasks (symbolic/non-symbolic numerical), along with physical magnitude estimation, graspability estimation, finger gnosis, and finger tapping tasks, would generate data conducive to encoding analysis. Such analyses can assist in characterizing the extent to which various BBPs are utilized in numerical tasks and how these processes align with sensorimotor tasks. The selection of these sensorimotor tasks can be tailored based on the theoretical claims being tested. For instance, past investigations into the interplay between fingers and numbers have focused on various facets of finger sensorimotor processing, including tapping, fine motor abilities, grasping, and gnosis. However, application of machine learning methods in this area has been limited to electrophysiological studies (Salehzadeh, Rivera, Man, Jalili, & Soylu, 2023; Salehzadeh, Soylu, & Jalili, 2023).

In the hypothetical study, ten different tasks across four categories would be used: 1) Finger/hand sensorimotor tasks—finger tapping, finger gnosis, and graspability; 2) Comparison tasks—physical magnitude, non-symbolic number (quantity), and symbolic (Arabic numeral); 3) Executive function tasks—working memory and cognitive control; and 4) Number processing—simple and complex arithmetic (Fig. 1). The finger tasks aim to capture sensorimotor patterns related to finger processing, while the comparison tasks aim to assess the extent to which physical magnitude, quantity, and symbolic comparison differ in their associations with the other tasks. The executive function tasks are used for comparison with the arithmetic tasks, helping evaluate their associations with sensorimotor and comparison tasks. Lastly, the inclusion of simple and complex arithmetic tasks aims to enable a comparison between retrieval and calculation strategies concerning their association with the finger sensorimotor tasks.

Data analysis would involve preprocessing and application of dimension reduction methods to create a reduced dimension version of the dataset. Decoding analysis would involve training of an algorithm (e.g., SVM, neural networks) to classify trial data into task categories. Feature importance analysis would reveal networks that most contribute to the classification. For example, SVM models that classify across the three comparison tasks (i.e., magnitude, non-symbolic, and symbolic) in a one-vs-rest fashion, would reveal voxel maps that uniquely contribute to classification of each comparison task. The decoding analysis can help identify sensorimotor networks dominant in the finger tasks and to what extent each of these contribute to the math tasks. For instance, a specific network predominantly manifesting in finger tapping, gnosis, and grasping, as well as in number comparison and addition, could be construed as a pivotal sensorimotor process linked to

finger sensorimotor processing, but also to the processing of numerical quantities. A broader identification and understanding of these central processes necessitate the accumulation of a corpus of research utilizing similar methodologies, meta-analysis studies aimed at pinpointing recurrent networks within analogous task contexts, alongside a taxonomy of functional networks (Uddin et al., 2023).

Data analytics methods applied to datasets with diverse tasks can also reveal how a single region contributes to many different functions. For example, the limited functions attributed to IPS and AG in numerical cognition models are partially due to the narrow task contexts considered. Functional fingerprint (Passingham et al., 2002) or functional diversity (Anderson et al., 2013) measures expose the one-to-many relationships between brain regions and tasks. These approaches facilitate the validation of claims about the relative and comparative engagement of an ROI across different tasks (Fig. 2). For instance, in the proposed hypothetical study, the functional fingerprint associated with the intraparietal sulcus (IPS) would be expected to show a stronger association with graspability compared to finger gnosis, or with magnitude comparison compared to complex arithmetic.

The proposed research design aims to test claims regarding the participation of the sensorimotor system in various number processes

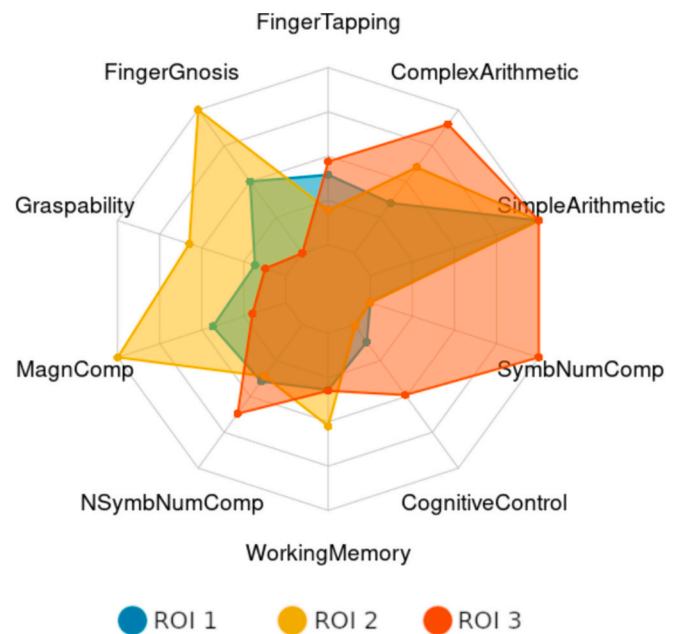


Fig. 2. Example functional fingerprints showing the association of three hypothetical ROIs with different tasks.

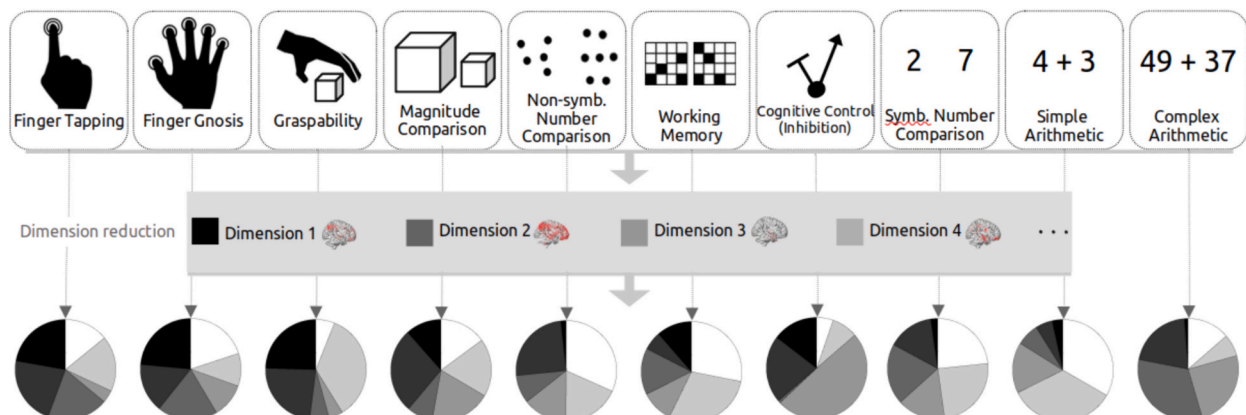


Fig. 1. Hypothetical paradigm design and identification of hidden dimensions.

and to examine the fundamental sensorimotor BBPs underpinning numerical cognition. The study presented here is purely hypothetical, illustrating the proposed approach. Progress in understanding human numerical abilities could be achieved through a series of studies examining the associations between various sensorimotor and number tasks and identifying fundamental processes consistently observed across these studies. Meta-analyses, compiling results from multiple studies could reveal sensorimotor patterns consistently linked with particular cognitive tasks. Yet, interpreting what these sensorimotor patterns mean will require theoretical progress. For instance, the simulation of sensorimotor states to facilitate cognitive skills has been suggested in various cognitive domains (e.g., Gallese & Sinigaglia, 2011; Gibbs, 2006; Pezzulo, Barca, Bocconi, & Borghi, 2010). Yet, we lack the terminology to articulate how simulation of sensorimotor states support cognition and the comprehensive understanding of the neural systems that dynamically emerge to mediate between the sensorimotor systems and the cognitive tasks. The hypothetical study outlined here is admittedly vague. Current guidelines and heuristics for utilizing data informatics approaches to identify fundamental sensorimotor processes involved in cognition are still underdeveloped. Nevertheless, these burgeoning techniques offer promising avenues that warrant further development and investigation.

5. Conclusion

Recently, cognitive neuroscience models have transitioned from localizationism to network-based approaches, where neural correlates of cognitive processes are understood as dynamic networks distributed throughout the brain. Further, the structure-function relations are described in *many-to-many* terms: a single function might engage multiple structures, while a single structure can be implicated in various functions. The rapid advancements in computational methods and the availability of extensive datasets have further enhanced our capacity to examine the brain's structural and functional networks. Yet, a disparity remains: The dominant cognitive ontology—the foundational concepts and associations used to study and model cognition—in numerical cognition, and more generally in cognitive neuroscience, still depends on the computational theory of mind, and does not lend itself to explain brain-body-environment relations.

Seeking a new cognitive ontology for numerical cognition requires developing new research methods and theoretical frameworks that are capable of dissecting our assumptions about mathematical abilities; by considering how a set of skills that evolved independently came together to enable our mathematical skills and how biological correlates of number processing are rooted in systems that support situated action. Data informatics approaches, such as machine learning, combined with expansive datasets afford novel ways to empirically probe theories on the role of the sensorimotor system in cognition.

I contend that our search should focus on identifying the foundational, building block processes (BBPs) within multimodal sensorimotor systems. These processes collaboratively function in a multitude of ways to facilitate cognitive abilities. Such an exploration necessitates innovative study designs where both the cognitive tasks and their related perceptuomotor tasks are jointly examined. The proposed approach to studying numerical cognition incorporates ideas on embodiment, evolutionary studies in biology and anthropology, cognitive science, and neuroscience, in addition to emerging data informatics methods in neuroimaging. Theories that are likely to come out of these efforts will explain not only how bodily systems that evolved to interact with a physical environment, build tools, and socially communicate can enable emergent skills like mathematics and reading, but also how different domains of cognition are interrelated by way of shared building block processes.

CRedit authorship contribution statement

Firat Soylu: Conceptualization, Writing – original draft, Writing – review & editing.

Declaration of competing interest

In regards to my manuscript, submitted to Acta Psychologica, I have NO affiliations with or involvement in any organization or entity with any financial interest (such as honoraria; educational grants; participation in speakers' bureaus; membership, employment, consultancies, stock ownership, or other equity interest; and expert testimony or patent-licensing arrangements), or non-financial interest (such as personal or professional relationships, affiliations, knowledge or beliefs) in the subject matter or materials discussed in the manuscript.

Data availability

No data was used for the research described in the article.

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